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Course: MPZ 3132-Engineering Mathematics IB   Model Answer No.02: Academic Year 2011/2012

1.

Let $y = \dfrac{1}{(D+\alpha)} f(x)$

Operating $(D+\alpha)$ for both sides

$(D+\alpha)y = f(x)$

$\dfrac{dy}{dx} + \alpha y = f(x)$

Integrating factor $= e^{\int \alpha dx} = e^{\alpha x}$

$e^{\alpha x}\dfrac{dy}{dx} + \alpha e^{\alpha x} y = f(x)e^{\alpha x}$

$\dfrac{d}{dx}(e^{\alpha x} y) = f(x)e^{\alpha x}$

Integrating

$e^{\alpha x} y = \int e^{\alpha x} f(x)dx$

$y = e^{-\alpha x}\int e^{\alpha x} f(x)dx$

$\therefore \dfrac{1}{(D+\alpha)} f(x) = e^{-\alpha x}\dfrac{1}{D} e^{\alpha x} f(x)$

1.1

1.1.1

$\dfrac{d^2y}{dx^2} + 5\dfrac{dy}{dx} + 6y = x^2e^{3x}$

$(D^2+5D+6)y = x^2e^{3x}$ $\dfrac{d}{dx} = D$

Auxiliary equation $\lambda^2 + 5\lambda + 6 = 0 \Rightarrow \lambda = -2$ or $\lambda = -3$

$\therefore$ Complementary function $y_c = Ae^{-2x} + Be^{-3x}$ where A and B are arbitrary constants.

Particular integral

$$y_p = \frac{1}{(D+3)(D+2)}x^2e^{3x} = \frac{(D+3)-(D+2)}{(D+3)(D+2)}x^2e^{3x} = \left(\frac{1}{(D+2)} - \frac{1}{(D+3)}\right)x^2e^{3x}$$

$$= \frac{1}{D+2}x^2e^{3x} - \frac{1}{D+3}x^2e^{3x}$$

$$= e^{-2x}\int e^{2x}x^2e^{3x}dx - e^{-3x}\int e^{3x}x^2e^{3x}dx = e^{-2x}\int x^2e^{5x}dx - e^{-3x}\int x^2e^{6x}dx$$

$$= e^{-2x}\left[\frac{1}{5}x^2e^{5x} - \frac{1}{25}2xe^{5x} + \frac{1}{125}2e^{5x}\right] - e^{-3x}\left[\frac{1}{6}x^2e^{6x} - \frac{2x}{36}e^{6x} + \frac{2}{216}e^{6x}\right]$$

$$= e^{3x}\left(\frac{x^2}{5} - \frac{2}{25}x + \frac{2}{125} - \frac{x^2}{6} + \frac{x}{18} - \frac{1}{108}\right) = e^{3x}\left(\frac{1}{30}x^2 - (\frac{2}{25} - \frac{1}{18})x + \frac{216-125}{125\times 108}\right)$$

$$= e^{3x}\left(\frac{x^2}{30} - \frac{(36-25)}{25\times 18}x + \frac{91}{13500}\right) = e^{3x}\left(\frac{x^2}{30} - \frac{11}{450}x + \frac{91}{13500}\right)$$

$$= \frac{e^{3x}}{13500}\left(450x^2 - 330x + 91\right)$$

$\therefore$ The general Solution is

$$y = Ae^{-2x} + Be^{-3x} + \frac{e^{3x}}{13500}(450x^2 - 330x + 91)$$

1.1.2

$$\frac{d^2y}{dx^2} - 7\frac{dy}{dx} - 12y = 3Sin5x$$

$\left(D^2 - 7D - 12\right)y = 3Sin5x$ where $\frac{d}{dx} = D$

Auxiliary equation $\lambda^2 - 7\lambda - 12 = 0 \Rightarrow \lambda = -3$ or $\lambda = 4$

$\therefore$ Complementary function $y_c = Ae^{-3x} + Be^{4x}$ Where A,B are arbitrary constants.

Particular integral

$$y_p = \frac{1}{(D-4)(D+3)}3Sin5x = \left(\frac{(D-3)-(D-4)}{(D-4)(D+3)}\right)3Sin5x = \left(\frac{1}{(D-4)} - \frac{1}{(D+3)}\right)3Sin5x$$

$$= \frac{1}{D-4}3Sin5x - \frac{1}{D+3}3Sin5x = 3\left[e^{4x}\int e^{-4x}Sin5xdx - e^{3x}\int e^{-3x}Sin5xdx\right]$$

$$= 3\left[e^{4x}\frac{e^{-4x}}{16+25}(-4Sin5x - 5Cos5x) - e^{3x}\frac{e^{-3x}}{9+25}(3Sin5x - 5Cos5x)\right]$$

$$= 3\left[\left(\frac{-4}{41} - \frac{3}{34}\right)Sin5x + \left(\frac{-5}{41} + \frac{5}{34}\right)Cos5x\right] = 3\left[\frac{(-136-123)}{1394}Sin5x + 5\frac{(41-34)}{1394}Cos5x\right]$$

$$y_p = 3\left(\frac{-37}{1394}Sin5x + \frac{5}{1394}Cos5x\right)$$

The general solution is

$$y = Ae^{-3x} + Be^{4x} + \frac{3}{1394}\left(-37Cos5x + 5Sin5x\right)$$

1.2

$$\frac{dx}{dt} - x - 3y = e^t$$

$$(D-1)x - 3y = e^t \text{.................(1)}$$

$$\frac{dy}{dt} - x + y = e^{4t}$$

$$(D+1)y - x = e^{4t} \text{.................(2)}$$

The equation (2) is operated by $(D-1)$

$$(D-1)(D+1)y - (D-1)x = (D-1)e^{4t} = 4e^{4t} - e^{4t}$$

$$(D^2-1)y - (D-1)x = 3e^{4t} \text{..............(3)}$$

$$(1)+(3) \quad (D^2-1)y - 3y = e^t + 3e^{4t}$$

$$(D^2-4)y = e^t + 3e^{4t}$$

Auxiliary equation $\lambda = \pm 2$

$\therefore$ Complementary function $y_c = Ae^{2t} + Be^{-2t}$

Particular integral

$$y = \frac{1}{(D-2)(D+2)}(e^t + 3e^{4t}) = \frac{1}{4}\left[\frac{(D+2)-(D-2)}{(D-2)(D+2)}\right](e^t + 3e^{4t})$$

$$y_p = \frac{1}{4}\left[\frac{1}{D-2} - \frac{1}{D+2}\right]\left(e^t + 3e^{4t}\right) = \frac{1}{4}\left[\frac{1}{D-2}\left(e^t + 3e^{4t}\right) - \frac{1}{D+2}\left(e^t + 3e^{4t}\right)\right]$$

$$= \frac{1}{4}\left[e^{2t}\int e^{-2t}\left(e^t + 3e^{4t}\right)dt - e^{-2t}\int e^{2t}\left(e^t + 3e^{4t}\right)dt\right]$$

$$= \frac{1}{4}\left[e^{2t}\left(-e^{-t} + \frac{3}{2}e^{2t}\right) - e^{-2t}\left(\frac{e^{3t}}{3} + \frac{3e^{6t}}{2}\right)\right]$$

$$= \frac{1}{4}\left[-e^t + \frac{3}{2}e^{4t} - \frac{e^t}{3} - \frac{e^{4t}}{2}\right] = \frac{1}{4}\left(\frac{-4}{3}e^t + \frac{8}{2}e^{4t}\right) = -\frac{1}{3}e^t + \frac{1}{4}e^{4t} = \frac{1}{4}\left(e^{4t} - \frac{4}{3}e^t\right)$$

General solution is

$$y = Ae^{2t} + Be^{-2t} + \frac{1}{4}(e^{4t} - \frac{4}{3}e^{t})$$

Operating equation (1) by $(D+1)$

$(D^2-1)x - 3(D+1)y = (D+1)e^{t}$.............(4)

$3\times(2) \quad 3(D+1)y - 3x = 3e^{4t}$.............(5)

$(5)+(4) \quad (D^2-4)x = 2e^{t} + 3e^{4t}$

Complementary function

$y_c = Ae^{2t} + Be^{-2t}$

Particular integral

$$y_p = \frac{1}{(D-2)(D+2)}(2e^{t} + 3e^{4t})$$

$$= \frac{1}{4}\left(\frac{1}{D-2} - \frac{1}{D+2}\right)(2e^{t} + 3e^{4t}) = \frac{1}{4}\left(\frac{1}{D-2}(2e^{t} + 3e^{4t}) - \frac{1}{D+2}(2e^{t} + 3e^{4t})\right)$$

$$= \frac{1}{4}\left[e^{2t}\int e^{-2t}(2e^{t} + 3e^{4t})dt - e^{-2t}\int e^{2t}(2e^{t} + 3e^{4t})dt\right]$$

$$= \frac{1}{4}\left[e^{2t}\int\left(2e^{-t} + 3e^{2t}\right)dt - e^{-2t}\int\left(2e^{3t} + 3e^{6t}\right)dt\right]$$

$$= \frac{1}{4}\left[e^{2t}\left(-2e^{-t} + \frac{3e^{2t}}{2}\right) - e^{-2t}\left(\frac{2e^{3t}}{3} + \frac{3e^{6t}}{6}\right)\right]$$

$$= \frac{1}{4}\left[-2e^{t} + \frac{3}{2}e^{4t} - \frac{2}{3}e^{t} - \frac{1}{2}e^{4t}\right]$$

$$= \frac{1}{4}\left[-\frac{8}{3}e^{t} + e^{4t}\right] = \frac{1}{12}\left(3e^{4t} - 8e^{t}\right)$$

The general solution

$$y_c = Ae^{2t} + Be^{-2t} + \frac{1}{12}(3e^{4t} - 8e^{t})$$

2.

2.1 $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+5y=58Cosx-4Sinx$

Given that

$y_T = ACosx + BSinx$

$\frac{dy_T}{dx} = -ASinx + BCosx$

$\frac{d^2y_T}{dx^2} = -ACosx - BSinx$

$-ACosx - BSinx + 2(-ASinx + BCosx) + 5(ACosx + BSinx) = 58Cosx - 4Sinx$

Equating the coefficients of $Cosx$

$-A+2B+5A=58$

$2B+4A=58$

$B+2A=29$.............(1)

Equating the coefficients of $Sinx$

$-B-2A+5B=-4$

$-2A+4B=-4$..............(2)

$(1)+(2)$ $5B=25$ $\therefore B=5$ $\therefore A=12$

2.2 $\therefore y_r = 12Cosx + 5Sinx$

Auxiliary equation $\lambda^2+2\lambda+5=0 \Rightarrow \lambda=-1\pm 2i$

Complementary function

$y_c = e^{-x}\left(A'Cos2x + B'Sin2x\right)$ Where A, B are arbitrary constants.

The general solution

$y = e^{-x}\left(A'Cos2x + B'Sin2x\right) + 12Cosx + 5Sinx$

2.3

$Z = xy$

$\frac{dz}{dx} = x\frac{dy}{dx} + y$........(1)

$$\frac{d^2z}{dx^2} = x\frac{d^2y}{dx^2} + \frac{dy}{dx} + \frac{dy}{dx}$$
$$\frac{d^2z}{dx^2} = x\frac{d^2y}{dx^2} + 2\frac{dy}{dx}.............(2)$$

$$x\frac{d^2y}{dx^2} + 2(x+1)\frac{dy}{dx} + (5x+2)y = 58Cosx - 4Sinx$$
$$\left(x\frac{d^2y}{dx^2} + 2\frac{dy}{dx}\right) + 2\left(x\frac{dy}{dx} + y\right) + 5xy = 58Cosx - 4Sinx$$
$$\frac{d^2z}{dx^2} + 2\frac{dz}{dx} + 5z = 58Cosx - 4Sinx$$

Using 2.2

$$Z = e^{-x}\left(A'Cos2x + B'Sin2x\right) + 12Cosx + 5Sinx$$
$$xy = e^{-x}\left(A'Cos2x + B'Sin2x\right) + 12Cosx + 5Sinx$$
$$y = \frac{1}{x}e^{-x}\left(A'Cos2x + B'Sin2x\right) + \frac{13}{x}\left(\frac{12}{13}Cosx + \frac{5}{13}Sinx\right)$$
$$y = \frac{1}{x}e^{-x}\left(A'Cos2x + B'Sin2x\right) + \frac{13}{x}\left(SinxCos\alpha + CosxSin\alpha\right)$$
$$y = \frac{1}{x}e^{-x}\left(A'Cos2x + B'Sin2x\right) + \frac{13}{x}Sin\left(x + \tan^{-1}\frac{12}{5}\right)$$

3.

3.1

$$\frac{d^2y}{dx^2} + 4y = 20e^{4x} \text{ Let } y_T = \alpha e^{4x}$$
$$\frac{dy_T}{dx} = 4\alpha e^{4x} \quad \frac{d^2y_T}{dx^2} = 16\alpha e^{4x}$$
$$16\alpha e^{4x} + 4\alpha e^{4x} = 20e^{4x}$$

$$e^{4x} \neq 0 \quad \therefore 20\alpha \quad \therefore \alpha = 1$$
$$\therefore y_T = e^{4x}$$

Auxiliary equation $\lambda^2 + 4 = 0 \Rightarrow \lambda = \pm 2i$

$\therefore$ Complementary function $y_c = A\,Cos2x + BSin2x$

3.2 $\therefore$ The general solution is $y = \left(ACos2x + BSin2x \right) + e^{4x}$

3.3

Given that

$$Z = x^3 y$$
$$\frac{dz}{dx} = 3x^2 y + x^3 \frac{dy}{dx}$$
$$\frac{d^2 z}{dx^2} = 6xy + 3x^2 \frac{dy}{dx} + 3x^2 \frac{dy}{dx} + x^3 \frac{d^2 y}{dx^2}$$
$$\frac{d^2 z}{dx^2} = x^3 \frac{d^2 y}{dx^2} + 6x^2 \frac{dy}{dx} + 6xy$$

$$x^3 \frac{d^2 y}{dx^2} + 6x^2 \frac{dy}{dx} + 2x\left(2x^2 + 3\right) y = 20e^{4x}$$
$$\left(x^3 \frac{d^2 y}{dx^2} + 6x^2 \frac{dy}{dx} + 6xy \right) + 4x^3 y = 20e^{4x}$$
$$\frac{d^2 z}{dx^2} + 4Z = 20e^{4x} \quad \therefore Z = \left(ACos2x + BSin2x \right) + e^{4x}$$
$$\therefore x^3 y = \left(ACos2x + BSin2x \right) + e^{4x}$$
$$\therefore y = \frac{1}{x^3}\left(ACos2x + BSin2x \right) + \frac{1}{x^3} e^{4x}$$

4.

4.1

$L\left(f(x) \right) = F(S) = \int_0^\infty e^{-sx} f(x)dx$ for $s \in \mathbb{C}$ is defined as the Laplace Trasformation of $f : \left[0, \infty\right) \to \mathbb{C}$

4.1.1

$$f(x) = \begin{cases} 0 & \text{if } 0 < x < 2 \\ 3x^2 & \text{if } 2 \le x \end{cases}$$

$$L\left(f(x) \right) = \int_0^\infty f(x)e^{-sx}dx = \int_0^2 0e^{-sx}dx + \int_2^\infty 3x^2 e^{-sx}dx = \int_2^\infty 3x^2 e^{-sx}dx$$

$$= 3\left[\frac{1}{-s}x^2e^{-sx} - \frac{1}{s^2}2xe^{-sx} - \frac{1}{s^3}2e^{-sx}\right]_2^{\infty} = \left[\frac{-4e^{-2s}}{s} - \frac{2.2e^{-2s}}{s^2} - \frac{2}{s^3}e^{-2s}\right]$$

$$= e^{-2s}\left(\frac{4}{s} + \frac{4}{s^2} + \frac{2}{s^3}\right) = \frac{-2e^{-2s}}{s^3}\left(2s^2 + 2s + 1\right)$$

4.1.2

$$f(x) = Sin^3x = \frac{1}{4}\left(3Sinx - Sin3x\right)$$

$$L\left(f(x)\right) = \int_0^{\infty}\frac{1}{4}\left(3Sinx - Sin3x\right)e^{-sx}dx = \frac{3}{4}\int_0^{\infty}Sinxe^{-sx}dx - \frac{1}{4}\int_0^{\infty}Sin3xe^{-sx}dx$$

$$L\left(f(x)\right) = \frac{3}{4}\left[\frac{e^{-3x}}{s^2+1}\left(-sSinx - Cosx\right)\right] - \frac{1}{4}\left[\frac{e^{-sx}}{s^2+9}\left(-sSin^2x - 3Cos3x\right)\right]$$

$$L\left(f(x)\right) = \left[\frac{-3e^{-sx}}{4(s^2+1)}\left(sSinx + Cosx\right) + \frac{e^{-sx}}{4(s^2+9)}\left(sSinx - 3Cosx\right)\right]_0^{\infty}$$

$$L\left(f(x)\right) = \frac{-3Cosx}{4(s^2+1)} - \frac{Cosx}{4(s^2+9)} = \frac{3}{4}\left(\frac{1}{s^2+9} - \frac{3}{s^2+1}\right)$$

4.2

$$L\left(f(x)\right) = F(s) = \int_0^{\infty}e^{-sx}f(x)dx$$

$$L\left(e^{ax}f(x)\right) = \int_0^{\infty}e^{-sx}e^{ax}f(x)dx = \int_0^{\infty}e^{-(s-9)}f(x)dx = F(s-9)$$

(1)

$$F(s) = \frac{2s+3}{s^2+4s+13} = \frac{2(s+2)-1}{(s+2)^2+3^2} = \frac{2(s+2)}{(s+2)^2+3^2} - \frac{1}{(s+2)^2+3^2}$$

$L(Sin3x) = \dfrac{3}{s^2+3}$ then $L\left(e^{-2x}Sin3x\right) = \dfrac{3}{(s+2)^2+3^2}$

$L(Cos3x) = \dfrac{s}{s^2+3^2}$ then $L\left(e^{-2x}Cos3x\right) = \dfrac{s+2}{(s+2)^2+3^2}$

$$\therefore L^{-1}\left(\frac{3}{(s+2)^2+3^2}\right)=e^{-2x}Sin3x$$

$$L^{-1}\left(\frac{s+2}{(s+2)^2+3^2}\right)=e^{-2x}Cos3x$$

$$L^{-1}\left(\frac{2s+3}{s^2+4s+13}\right)=L^{-1}\left(\frac{2(s+2)}{(s+2)^2+3^2}-\frac{1}{(s+2)^2+3^2}\right)$$

$$=2L^{-1}\left(\frac{s+2}{(s+2)^2+3^2}\right)-\frac{1}{3}L^{-1}\left(\frac{3}{(s+2)^2+3^2}\right)=2e^{-2x}Cos3x-\frac{1}{3}e^{-2x}Sin3x$$

$$F(s)=\frac{s^2+4}{(s+1)(s^2+2)}$$

$$\frac{s^2+4}{s+1}=\frac{A}{s+1}+\frac{Bs+c}{s^2+2}$$

$$s^2+4=A(s^2+2)+(Bs+C)(s+1)$$

$$s=-1 \quad 3A=5 \qquad \therefore A=\frac{5}{3}$$

$$1=A+B \quad \therefore B=1-\frac{5}{3}=\frac{-2}{3}$$

$$4=2A+C \quad \therefore C=4-\frac{10}{3}=\frac{2}{3}$$

$$\frac{S^2+4}{(S^2+2)(S+1)}=\frac{5}{3}\frac{1}{(S+1)}+\frac{(-2S+2)}{S^2+2}$$

$$=\frac{5}{3}\frac{1}{S+1}-2\frac{S}{S^2+2}+2\frac{1}{S^2+2}$$

$$L^{-1}\frac{S^2+4}{(S+1)(S^2+4)}=\frac{5}{3}L^{-1}\left(\frac{1}{S+1}\right)-2L^{-1}\frac{S}{S^2+(\sqrt{2})^2}+\frac{2}{\sqrt{2}}L^{-1}\frac{\sqrt{2}}{S^2+(\sqrt{2})^2}$$

$$=\frac{5}{3}e^{-x}-\frac{2}{3}Cos\sqrt{2}x+\frac{\sqrt{2}}{3}Sin\sqrt{2}x$$

5.

$$L\left(x^n e^{-\alpha x}\right) = \int_0^\infty x^n e^{-\alpha x} e^{-sx} dx = \int_0^\infty x^n e^{-(\alpha+s)x} dx$$

$$= \left[\begin{array}{l} \frac{-1}{(s+\alpha)} x^n e^{-(s+\alpha)x} - \frac{1}{(s+\alpha)^2} n x^{n-1} e^{-(s+\alpha)x} - \frac{1}{(s+\alpha)^3} n(n-1) x^{n-2} e^{-(s+\alpha)x} \\ -\frac{1}{(s+\alpha)^4} n(n-1)(n-2) x^{n-2} \ldots \frac{n(n-1)(n-2)\ldots 2}{(s+\alpha)^n} e^{-(s+\alpha)x} - \frac{n!}{(s+\alpha)^{n+1}} e^{-(s+\alpha)x} \end{array} \right]_0^\infty$$

$$x \xrightarrow{\lim} \infty \; x^n e^{-(s+\alpha)} x = 0 \quad \forall n \in \mathbb{Z}^+$$

$$= \frac{n!}{(S+\alpha)^{n+1}}$$

5.1 $\quad \frac{d^2 y}{dx^2} - 6\frac{dy}{dx} + 9y = x^4 e^{3x} \quad y(0) = 2 \left(\frac{dy}{dx}\right)_{x=0} = 6$

Taking the Laplace transformation

$$S^2 Y(s) - Sy(0) - y^{'}(0) - 6\left(SY(S) - y(0)\right) + aY(S) = L\left(x^4 e^{3x}\right)$$

$$Y(s)\left(S^2 - 6S + 9\right) - 2S - 6 + 12 = \frac{4!}{(S-3)^{4+1}}$$

$$\left(S-3\right)^2 Y(S) = \frac{4!}{\left(S-3\right)^5} + 2\left(S-3\right)$$

$$Y(S) = \frac{4!}{(S-3)^7} + \frac{2}{(S-3)}$$

Taking inverse laplace transformation

$$y(s) = L^{-1}\left(\frac{4!}{(S-3)^7} + \frac{2}{(S-3)}\right)$$

$$= L^{-1}\frac{4!}{(S-3)^7} + 2L^{-1}\left(\frac{1}{(S-3)}\right)$$

$$= \frac{1}{5\times 6} L^{-1}\frac{6!}{\left(S-3\right)^{6+1}} + 2L^{-1}\left(\frac{1}{(S-3)}\right)$$

$$= \frac{1}{30} x^6 e^{3x} + 2e^{3x}$$

$$y(s) = \frac{1}{30} e^{3x}\left(x^6 + 60\right)$$

$x(0) = -1 \ \ y(0) = 0$

$$\frac{dx}{dt} = 6x - 3y + 8e^{t}$$

$$\frac{dy}{dt} = 2x + y + 4e^{t}$$

Let $L(y(t)) = y \quad L(x(t)) = x$

Taking the Laplace transformation

$$SX - x(0) = 6X - 3Y + 8\frac{1}{S-1}$$

$$(S-6)X + 3Y = \frac{s}{S-1} - 1 = \frac{-S+9}{S-1} \ldots\ldots\ldots\ldots\ldots(1)$$

$$SY - y(0) = 2X + Y + 4\frac{1}{S-1}$$

$$\left(S-1\right)Y - 2X = \frac{4}{S-1} \ldots\ldots\ldots\ldots\ldots(2)$$

From (1) $X + \dfrac{3}{S-6}Y = \dfrac{-S+9}{\left(S-1\right)\left(S-6\right)} \ldots\ldots\ldots\ldots(A)$

From (2) $\dfrac{\left(S-1\right)}{2}Y - X = \dfrac{2}{S-1} \ldots\ldots\ldots\ldots(B)$

(A)+(B) $\left(\dfrac{S-1}{2} + \dfrac{3}{S-6}\right)Y = \dfrac{-S+9}{\left(S-1\right)\left(S-6\right)} + \dfrac{2}{\left(S-1\right)}$

$$\frac{\left(S^{2} - 7S + 12\right)Y}{2\left(S-6\right)} = \frac{-S+9+2S-12}{\left(S-1\right)\left(S-6\right)}$$

$$Y = \frac{2\left(S-3\right)\left(S-6\right)}{\left(S-1\right)\left(S-6\right)\left(S-3\right)\left(S-4\right)}$$

$$Y = \frac{2}{\left(S-1\right)\left(S-4\right)} = \frac{-2/3}{\left(S-1\right)} + \frac{2/3}{\left(S-4\right)}$$

Taking inverse Laplace transformation

$$y(t) = \frac{-2}{3}L^{-1}\left(\frac{1}{S-1}\right) + \frac{2}{3}L^{-1}\left(\frac{1}{S-4}\right) = \frac{-2}{3}e^{t} + \frac{2}{3}e^{4t} = \frac{2}{3}\left(e^{4t} - e^{t}\right)$$

$$\frac{(1)}{3} \quad \frac{\left(S-6\right)X}{3}+Y=\frac{-S+9}{3\left(S-1\right)}.................(\alpha)$$

$$\frac{(2)}{S-1} \quad y-\frac{2}{\left(S-1\right)}X=\frac{4}{\left(S-1\right)^2}...............(\beta)$$

$$(\alpha)-(\beta) \quad \left(\frac{S-6}{3}+\frac{2}{S-1}\right)X=\frac{-S+9}{3(S-1)}-\frac{4}{(S-1)^2}$$

$$\frac{\left(S^2-7S+12\right)X}{3(S-1)}=\frac{-S^2+S+9S-9-12}{3(S-1)^2}$$

$$X=\frac{-\left(S^2-10S+21\right)}{(S-1)(S-4)(S-3)}=\frac{-(S-7)(S-3)}{(S-1)(S-9)(S-3)}=\frac{7-S}{(S-1)(S-4)}$$

$$\frac{7-S}{(S-1)(S-4)}=\frac{A}{(S-1)}+\frac{B}{(S-4)}$$

$$A=-2$$
$$B=1$$

$$X=\frac{-2}{S-1}+\frac{1}{S-4}$$

Taking inverse laplace transformation

$$x(t)=-2L^{-1}\left(\frac{1}{S-1}\right)+L^{-1}\left(\frac{1}{S-4}\right)=-2e^{t}+e^{4t}$$
$$x(t)=e^{4t}-2e^{t}$$